Name_____ Date_____ Partners _____

GEOMETRICAL OPTICS

OBJECTIVES

- To examine Snell's Law and observe total internal reflection.

- To understand and use the lens equations.

- To find the focal length of a converging lens.

- To discover how lenses form images.

- To observe the relationship between an object and the image formed by a lens.

- To discover how a telescope works

OVERVIEW

Light is an electromagnetic wave. The theory of the propagation of light and its interactions with matter is by no means trivial; nevertheless, it is possible to understand most of the fundamental features of optical instruments such as eyeglasses, cameras, microscopes, telescopes, etc. through a simple theory based on the idealized concept of a light ray.

A **light ray** is a thin pencil of light that travels along a straight line until it encounters matter, at which point it is *reflected*, *refracted*, or *absorbed*. The thin red beam from a laser pointer is a good approximation of such a ray. The study of light rays leads to two experimental observations:

1. When a light ray is reflected by a plane surface, the angle of reflection θ_2 equals the angle of incidence θ_1, as shown in Fig. 1.
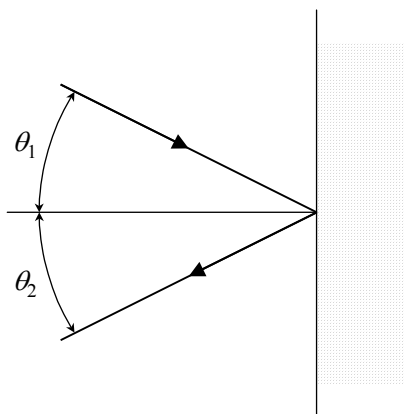

Fig. 1 Reflection: $\theta_1 = \theta_2$

2. When a light ray travels from one transparent medium into another, as shown in Fig. 2, the ray is generally "bent" (refracted). The directions of propagation of the incident and refracted rays are related to each other by **Snell's law**:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \tag{1}$$

where the dimensionless number **n** is called the index of refraction and is characteristic of the material.
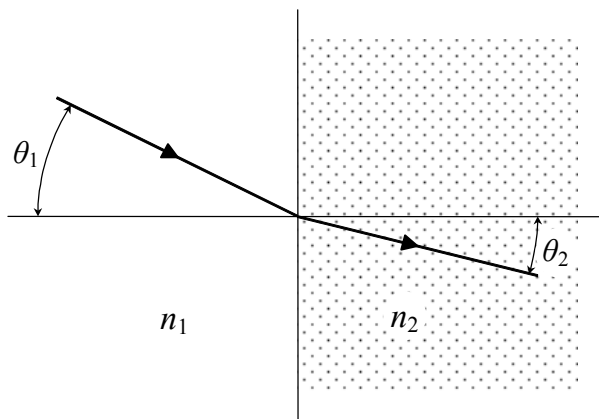

Fig. 2. Refraction: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

Note that if $n_1 \sin \theta_1 > n_2$, no solution is possible for $\sin \theta_2$. In this case, none of the light will pass through the interface. *All* of the light will be reflected. This **total internal reflection** is more perfect than reflection by any metallic mirror.

Most transparent materials have indices of refraction between 1.3 and 2.0. The index of refraction of a vacuum is by definition unity. For most purposes, the index of refraction of air can also be taken as unity ($n_{air} = 1.0003$)

Accurate measurements show the index of refraction to be a function of the wavelength and thus of the color of light. In general, one finds that:

$$n_{blue} > n_{red} \tag{2}$$

A simplified theoretical explanation of these observations is given by **Huygens' Principle**, which is discussed in elementary physics texts.

INVESTIGATION 1:   SNELL'S LAW

Note: The room lights will be turned out for these investigations. It will sometimes be difficult to read and write in this manual. Use the desk lamp as needed and good luck. Be patient!

In this investigation, you will observe and verify Snell's Law by using both a rectangular block and a prism. You will also observe total internal reflection in a prism.

You will need the following materials:

- Index of refraction block made of Lucite

- Triangular prism made of Lucite

- Protractor

- Light ray box

- Graphing paper from roll (approximately 40 cm for **Activity 1-1** and 40 cm for **Activity 1-2**).

Activity 1-1: Verifying Snell's Law
In this activity, you will verify Snell's law by using the light ray box with a single ray and the rectangular plastic block.

PLEASE TAKE CARE NOT TO SCRATCH THIS BLOCK OR THE OTHER OPTICAL ELEMENTS!
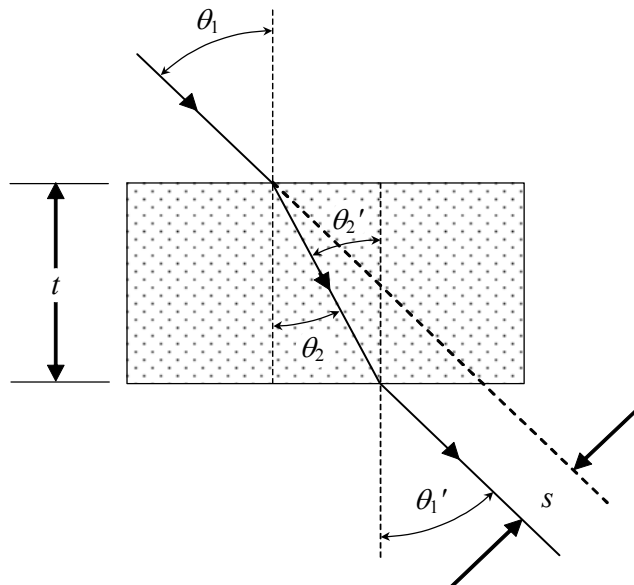

Fig. 3. Plate with parallel surfaces.

1. Place the block on a piece of graph paper and align it with the grid. Make sure to place it so that it has at least 10 cm of graph paper on either side. It may help to tape the paper to the table. Trace the outline of the block on the graph paper.

Note: Only one diagram will be drawn for each group. There are at least three activities that have you draw light rays, so make sure every student does at least one ray tracing diagram.

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2. Using the single aperture mask, let a single ray from the ray box fall at an oblique angle on the plastic block as in Fig. 3.

Hint: Larger values of θ_1 produce better results.

3. Mark on the graph paper the entry and exit points of the light. Also mark points on the incident and exit rays far from block. This will be necessary to determine the angles.

4. After removing the block, trace the light ray paths and use the protractor to measure the angles θ_1, θ_2, $\theta_1{}'$ and $\theta_2{}'$. Extrapolate the incident ray so that you can measure s, the "shift" of the output ray relative to the incident ray. Record your results in **Table 1-1**.

Note: We have used the subscript 1 for air and the subscript 2 for plastic, regardless of the direction of the ray. Other conventions are equally valid.

Table 1-1

θ_1	θ_2	$\theta_1{}'$	$\theta_2{}'$	s

5. Use Snell's Law to determine the index of refraction n for the block (assume $n_{air} = 1$).

n _____

Question 1-1: Is $\theta_1 = \theta_1{}'$ satisfied? Are the incident and exit rays parallel? Explain how your measurements support your answers.

Question 1-2: From Fig. 3 we can see that:

$$s = t\sin(\theta_1 - \theta_2)/\cos\theta_2 \qquad (3)$$

What does this tell you about the path of a ray through the *center* of a **thin** lens? (Hint: Imagine your Lucite block getting thinner and think about how the offset between the incident and exit rays would change.)

Activity 1-2: Light Passing Through a Prism

In this activity, you will study the propagation of light through a prism, as well as observe total internal reflection.
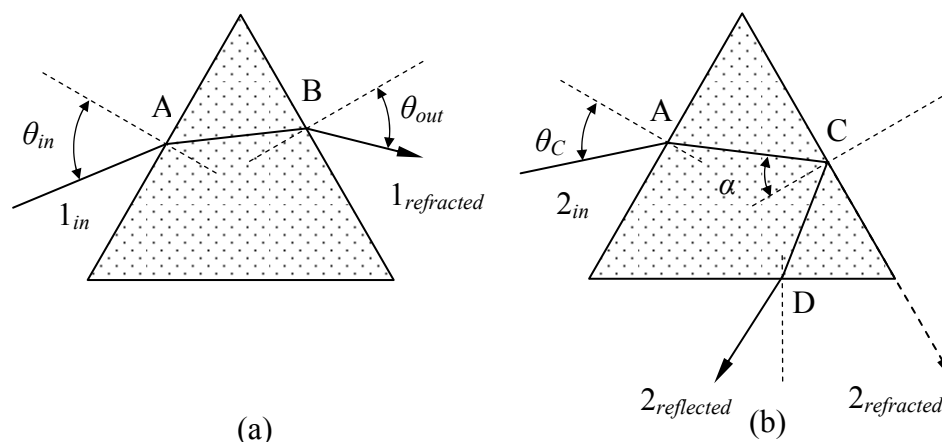

Fig. 4. Refraction and total reflection in a prism.

Two examples of light propagation in a prism are shown in Fig. 4. As you will recall, at each surface some of the light is reflected and some of the light is refracted. Fig. 4a shows a light ray entering the prism at A at an angle θ_{in} (relative to the normal), the refracted ray at the front surface, and the refracted ray at the rear surface leaving the prism at B at an angle θ_{out}. [Note: At each interface there will also be a reflected ray, but, for clarity, we don't show them here.]

Fig. 4b shows the case where the refracted output ray would come out along the edge of the surface ($\theta_{out} = 90°$). Any angle smaller than θ_C will produce light that hits the rear surface so that $n \sin \alpha \geq 1$, the condition for *total internal reflection*.

1. To observe this total internal reflection, a triangular prism will be used. Place the prism on a clean area of the graph paper or a new sheet.

2. Set the ray box so a single light ray falls on one side of the prism.

3. Vary the entrance angle of the ray by slowly rotating the prism. Note that there is a point at which no light is refracted out. Mark the positions of the rays when this *total internal reflection* occurs, as well as trace around the prism. Make sure to mark the incident ray, the point at which this ray strikes the back of the prism, and the reflected ray once it has exited the prism.

4. From these markings and using the protractor, find the internal reflection angle α. [Hint: Extend lines AC and CD and measure the included angle, which is 2α]

α _____

Question 1-3: Use Snell's Law to derive an equation for n in terms of α. Show your work and calculate n.

n _____

5. Slowly rotate the prism again and note that the exiting ray spreads out into various colors just before total internal reflection occurs.

Question 1-4: Why does the light spread into different colors prior to total internal reflection?

6. Start at the point of total internal reflection and rotate the prism *slightly* to increase the entrance angle. You should see a weak ray just grazing along the outside of the prism base. Note very carefully which color emerges outside the prism first (red or blue).

Question 1-5: What does this tell you about the relative magnitudes of n_{red} and n_{blue} for Lucite?

INVESTIGATION 2: CONVERGING LENSES

Most optical instruments contain lenses, which are pieces of glass or transparent plastics. To see how optical instruments function, one traces

light rays through them. We begin with a simple example by tracing a light ray through a single lens.

We apply Snell's law to a situation in which a ray of light, coming from a medium with the refractive index $n_1 = 1$, e.g. air, falls onto a glass sphere with the index $n_2 = n$, shown in Fig. 5.
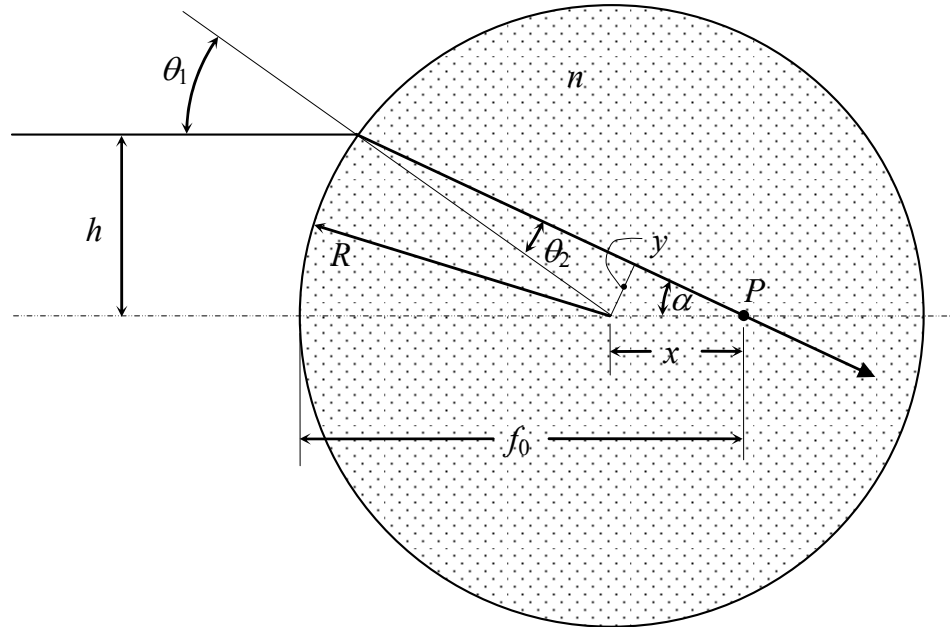
Fig. 5 Spherical lens.

We have in that case

$$\sin\theta_1 = n\sin\theta_2 \qquad (4)$$

Using the law of sines and Fig. 5, we obtain

$$\frac{\sin\alpha}{\sin\theta_2} = \frac{y/x}{y/R} = \frac{R}{x} \qquad (5)$$

or

$$x = R\frac{\sin\theta_2}{\sin\alpha}. \qquad (6)$$

If we make the simplifying assumption $h \ll R$, we can use the approximation

$$\sin\alpha = \frac{h}{\sqrt{(R+x)^2 + h^2}} \approx \frac{h}{R+x}. \qquad (7)$$

This yields

$$x \approx R\sin\theta_2 \frac{(R+x)}{h} = \frac{R(R+x)\sin\theta_1}{nh} = \frac{R(R+x)h}{nhR}$$

or

$$nx \approx R+x, \tag{8}$$

where $R+x$ is the distance from the front of the sphere to the point P where the ray crosses the **optical axis**. We call this distance the **focal length** f_0. Setting $f_0 = R+x$, Eq. (8) now reads:

$$n\left(f_0 - R\right) \approx f_0, \text{ or } f_0 \approx \frac{nR}{n-1} \tag{9}$$

This is a remarkable result because it indicates that, within the limits of our approximation $h << R$, the focal length f_0 is independent of h. This means that all rays that come in parallel to each other and are close to the axis are collected in one point, the **focal point**, F.

Note that our simple theory of a lens applies only to those cases in which the focal point is inside the sphere. A lens whose focal point is on its inside is not very useful for practical applications; we want it to be on the outside. (Actually, whether inside or out, spheres, for various reasons, do not make very useful lenses.) We will therefore study a more practical lens, the **planoconvex** lens. This lens is bounded on one side by a spherical surface with a radius of curvature R and on the other by a plane (see Fig. 6). To keep things simple we make the additional assumption that it is very thin, i.e. that $d << R$. Now we trace an arbitrary ray that, after having been refracted by the spherical front surface, makes an angle θ_1 with the optical axis, as shown in Fig. 6.
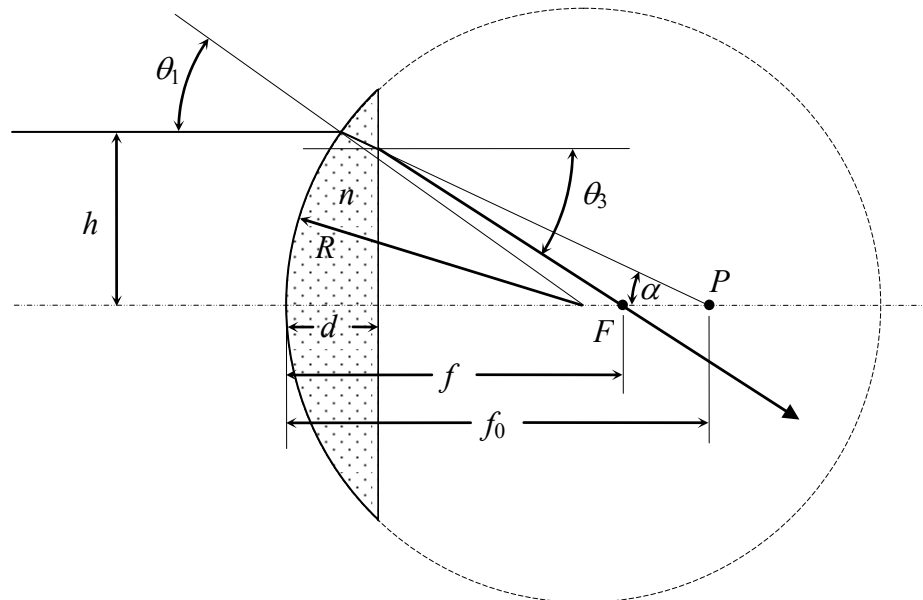

Fig. 6 Focal point of planoconvex lens.

If there were still a full glass sphere, this ray would intersect the optical axis at the point P, a distance f_0 from the front surface. On encountering the planar rear surface of the lens it will instead, according to Snell's law, be bent to intercept the axis at the point F, a distance f from the front. Behind the rear surface is air, so, on the encounter with the second surface Snell's law becomes:

$$n\sin\alpha = \sin\theta_3 \tag{10}$$

But

$$\sin\theta_3 \approx \frac{h}{f} \text{ and } \sin\alpha \approx \frac{h}{f_0} \tag{11}$$

Hence,

$$f \approx \frac{f_0}{n}, \tag{12}$$

i.e. in this case the distance f is independent of the distance h (as long as $h << R$ and $d << R$). Using Eq. (9) in Eq. (12) we find that all incoming rays that are parallel to the optical axis of a *thin* planoconvex lens are collected in a focal point at a distance

$$f = \frac{R}{n-1} \tag{13}$$

behind the lens.

What about rays that are not parallel? One can show that all rays issuing from the same object point will be gathered in the same image point (as long as the object is more than one focal length away from the lens).

To find the image point, we only need consider two rays (we'll discuss three that are easy to construct) and find their intersection.

Let us assume that there is a point source of light at the tip of object O at a distance $o > f$ in front of the lens. Consider the three rays issuing from this source shown in Fig. 7:
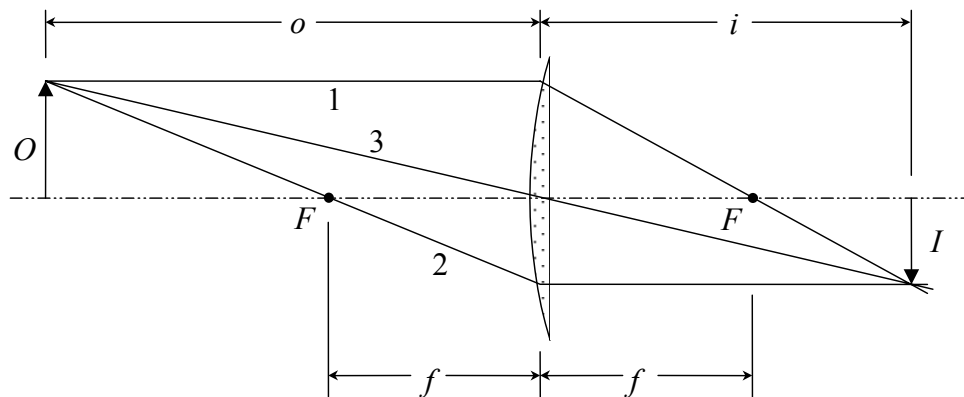

Fig. 7   Image construction.

1. **A ray that is parallel to the axis**. According to what we have just learned, it will go through the focal point F behind the lens.

2. **A ray that goes through the focal point F in front of the lens**. With a construction analogous to the one shown in Fig. 6, one can show that light parallel to the axis coming from behind the lens will go through the focal point in front. Our construction is purely geometrical and cannot depend on the direction of the light beam. We conclude that light that passes through the focal point in front of the lens must leave the lens parallel to the axis. This ray will intersect the first ray at the tip of image I at a distance i behind the lens.

3. **A ray that goes through the center of the lens**. At the center, the two glass surfaces are parallel. As we have seen, light passing through such a plate will be shifted by being bent towards the normal at the first interface and then back to the original direction at the second interface. If the plate (in our case the lens) is thin, the shift will be small. We assumed our lens was very thin, so we can neglect any such shift.

From Fig. 7 it should not be difficult for you to see (from "similar triangles") that:

$$\frac{I+O}{o} = \frac{I}{f} \text{ and } \frac{I+O}{i} = \frac{O}{f}$$

Hence we arrive at the following *thin lens equations*:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{14a}$$

$$\frac{i}{I} = \frac{o}{O} \tag{14b}$$

We define **magnification** M to be the ratio of the image size I to the object size O:

$$M \equiv \frac{I}{O} \tag{15a}$$

or (by application of Eqs. 14):

$$M = \frac{f}{o-f} \tag{15b}$$

The image in Fig. 7 is called a ***real*** image because actual rays converge at the image. The method of image construction used in Fig. 7, as well as thin lens formulae, can also be formally applied to situations where that is not the case.

What about when the object is *closer* than one focal length? In Fig. 8, an object O is placed within the focal distance ($o < f$) of a lens. Following the usual procedure, we draw the ray going through the center of the lens and the one that is parallel to the axis. We add a third ray, originating from O but going in a direction as if it had come from the first focal point F. All these are real rays and we draw them as solid lines. We extend the three lines backward as dashed lines and note that all three meet in a single point Q in front of the lens. To an observer behind the lens, the light coming from O will *seem* to come from Q and an upright, magnified image of the object O will be seen. *This image is a **virtual** image and not a **real** image since no light actually issues from Q.*
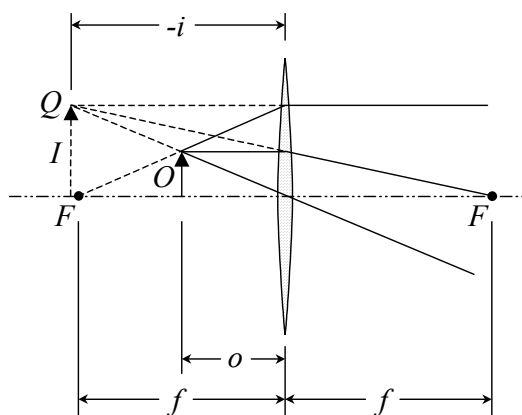

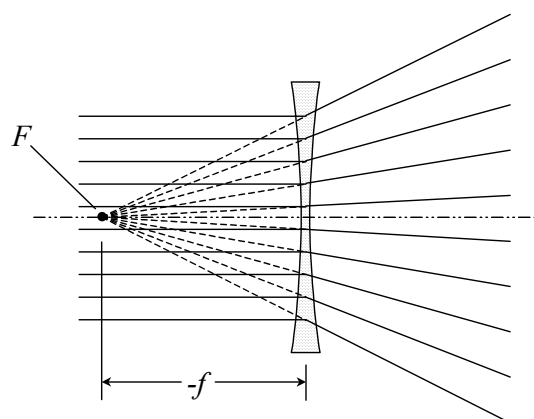Fig. 8 Magnifying glass.   Fig. 9 Biconcave lens.

By an appropriate choice of notation convention, we can apply the thin lens formulae to the magnifying glass. By way of a specific example, setting $o = f / 2$ in these equations, for instance, yields $i = -f$, $I = -2O$ and $M = -2$. We interpret the minus sign in the first equation as meaning that i extends now in *front* of the lens and the minus sign in the second that the image is no longer inverted but upright. We therefore introduce the following convention: o and i are taken to be positive if the object is to one side and the image on the other side of the lens. O and I are taken to be positive if the object is upright and the image is inverted.

We can carry this one step further. Concave lenses (lenses that are thinner in the center than on the rim) make parallel incident light diverge. We formally assign to them a negative focal length. Fig. 9[1] shows that this is justified. To an observer behind such a lens, the incident parallel rays do seem to have come from a virtual focal point in front of the lens.

In this investigation, you will familiarize yourself with a converging lens. You will first find the focal length of the lens and then observe how such a lens creates an image.

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[1]The lens shown in Fig. 9 is a **biconcave** lens; Eqs. 14 apply to it as well, as long as it is **thin.**

For this investigation, you will need the following materials:

- Planoconvex lens made of Lucite

- Light ray box with five ray pattern

- 40 cm of graph paper from roll for focal length activity

- 60 cm of graph paper from roll for ray tracing

Activity 2-1: Finding the Focal Length

1. Place the ray box on top of the piece of graph paper. Select the five ray pattern by replacing the end piece.

2. In order to effectively do this activity, the rays must enter the lens parallel to one another. To adjust the rays, slowly move the top of the box until the rays are parallel with the lines on the graph paper.
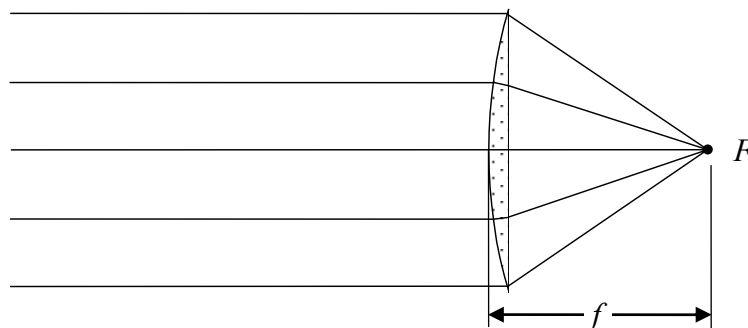

Fig. 10 Focal point of a planoconvex lens.

3. Place the lens on the center of your graph paper. Let the center ray from the ray box pass through the center of the lens at a 90º angle, as shown in Fig. 10. Trace the position of the lens on the graph paper.

4. Note that the rays converge at a point on the other side of the lens. This is the focal point F for the lens. To measure it, make points that will allow you to trace the rays entering and leaving the lens.

5. Remove both the lens and the ray box to measure f.

f _____ cm

Prediction 2-1: What will happen if you place the lens backwards over the position in steps 3 and 4? What will happen to the focal point *F* and focal length *f* ?

6. Turn the lens around and place it at the previous position to determine if the orientation of the lens influences its focal length.

Question 2-1: Do the lines converge at the same point as the value that you found in step 4? Should light incident on either side collect at the same point? What does this tell you about the lens?

Activity 2-2: Ray Tracing

This activity is designed to test the imaging properties of the lens. A ray-tracing diagram like the one shown in Fig. 7 will be created.

1. Place a clean 60 cm long piece of graph paper on the table.

2. Align the planoconvex lens somewhere on the graph paper. Allow about 25 cm clear on either side of the lens. Draw the central axis (see Fig. 7). Draw around the lens to mark its position and mark the two focal points *F* on the central axis on either side of the lens. Use the value you found in **Activity 2-1**.

3. To test how an image is formed, you will draw an object arrow like that shown in Fig. 7 on your piece of graph paper. Place the tip of the arrow at a distance of 2*f* from the lens and about 1.5 cm from the central axis. Record your values for the object in **Table 2-1**.

4. Using a single ray from the ray box, mark on your paper the ray paths on both sides of the lens the rays shown in Fig. 7. Use a different marking scheme (•×o) for points along each of the three rays. Mark two points on either side of the lens to help you draw the rays later after you remove the lens. Your three rays should be as follows:

- Ray 1 should go through *F* on its way to the image point

- Ray 2 should enter the lens parallel to the optical axis

- Ray 3 should pass through the lens nearly unbent

5. Note where the three rays seem to indicate the image should be. You have found the image of only one point – the tip of the object arrow, but that is enough to deduce the entire image. Draw an arrow indicating where the image is. Measure the image distance and image size and fill in the experimental values in **Table 2-1**. Include your sheet with your group report.

6. Based on the height and position of your object in **Table 2-1**, use the thin lens formulae (Eqs. 14) to calculate the theoretical values for the image distance and size. Use the focal length *f* you measured above. Insert your calculated values in **Table 2-1**.

Table 2-1

	Object distance (i)	Object size (I)	Image distance (o)	Image size (O)
Experimental				
Theory	XXXXXXXXXXXXXXX			

Question 2-2: Compare your experimental and theoretical values for the image distance and size in **Table 2-1**. Discuss the agreement. *Do not be disappointed if things do not work out exactly. Remember that you are not using a truly thin lens.*

INVESTIGATION 3: IMAGE FORMATION BY CONVERGING LENSES

In order to examine the image formed by a converging lens, you will need the following:

- Optical bench
- 100 mm lens[2]
- illuminated object
- small see-through ruler
- small desk lamp

- lens holder
- 200 mm lens
- viewing screen
- 3 meter tape

Activity 3-1: Image Formation by a Converging Lens

In this activity, you will see the relative positions for the object and image distances formed by a converging lens.
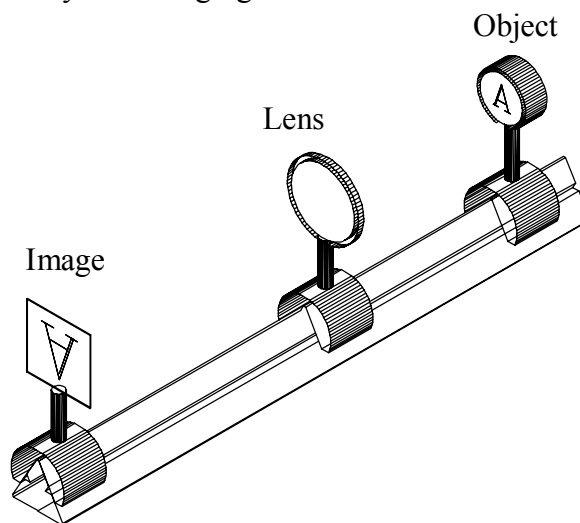

Fig. 11 Creation of an inverted real image on the optical bench.

Prediction 3-1: If the object is always outside of the focal point, do you expect the image distance to increase or decrease if the object distance is increased?

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[2] Lenses are labeled by focal length, not by any geometrical parameters such as a radius of curvature.

Prediction 3-2: What do you expect will happen to the image size if the object distance is increased?

1. Place an illuminated object together with the mounted 100 mm lens and the viewing screen on the optical bench as shown in Fig. 11.

2. Measure the size of the object, using the small ruler.

    Object size: _____ cm

3. Set the initial object distance to 15 cm.

4. Find the location of the image. To do this, move the screen until a sharp image is formed. Record the image distance, as well as the image size in the second two columns of **Table 3-1**.

Table 3-1 Experimental Data

Object Distance	Image Distance (cm)	Image Size (cm)	Magnification	Upright or Inverted?	Image: Real or Virtual?
15 cm					
20 cm					
30 cm					

5. Calculate the magnification of your image and record in **Table 3-1**.

6. Is the image upright or inverted? Real or virtual? Record your observation in **Table 3-1**.

7. Try two other object distances, 20 cm and 30 cm. Record the image distance, image size, magnification, orientation and image properties of the image in **Table 3-1**.

Table 3-2 Theoretical Results

Object distance	Image distance (cm)	Image size (cm)	Magnification
15 cm			
20 cm			
30 cm			

8. Use the thin lens formulae to calculate the image distance, image size, and magnification for the three object distances shown in **Table 3-2**. Maybe each lab partner can calculate one each. Enter your calculated values into the table.

Question 3-1: How good is the agreement between your experimental data in **Table 3-1** and your calculations in **Table 3-2**? Compare with your Predictions 3-1 and 3-2.

Prediction 3-3: All of the previous measurements involved an object distance greater than the focal length. What will the image look like if the object distance is less than the focal length?

9. Make sure that the object is oriented so it is facing the center of the room and at the end of the optical bench furthest away from the end of the table. This will make your upcoming observations significantly easier.

10. Place the 10 cm lens so that the object distance is approximately 5-8 cm.

11. Stand at the end of the table so you are looking through the lens at the object. Your distance to the lens should now be approximately 1 m.

Question 3-2: Describe your image. Is it upright or inverted? If you were to put a screen where you are looking, would an image form there? What does this tell you about the image? Is it real or virtual?

12. Now have one of the students in the group slowly move the lens away from the object until it is approximately 10 cm from the object. Make sure that another student is standing at the end of the optical bench still looking through the lens. Continue looking until the image disappears.

Question 3-3: Why is it that when the object is at the focal length it produces no image? (Hint: consider the thin lens formulae).

Activity 3-2: Test Fixed Distance

1. In your prelab, you were asked to assume that the object and viewing screen were placed a fixed distance $C = i + o = 50$ cm apart, and then to find two positions of the lens that will produce an image on the screen. Use the values for i and o that you or your partners determined and record the distances in **Table 3-3** below.

Table 3-3

		Object distance (cm)	Image distance (cm)	Object size (cm)	Image size (cm)
Prelab	1			XXXXXXX	XXXXXXX
	2			XXXXXXX	XXXXXXX
Experiment	1				
	2				

2. Test your values by positioning the screen 50 cm from the object. Find the two lens positions that produce sharp images. Measure and record

(in **Table 3-3**) the image and object distance and size. Do this for both values of your object position.

Question 3-4: Do your values agree within error with those in the prelab? What could be a source of any error?

Activity 3-3: Simulating a Camera

1. Place the object at one end of the optical bench and the viewing screen at the other end.

2. Place the 100 mm lens near the viewing screen and move the lens until you see a focused image on the screen. (On a real camera, a focus knob will move the lens elements toward or away from the film.) Note the size of the image.

3. Repeat with the 200 mm lens (a "telephoto lens" in photography language). Is the image larger?

Question 3-5: Based on these results, why are telephoto lenses so long?

Activity 3-4: A Telescope

In this activity, you will see how converging lenses are used in the formation of telescopes.

1. This setup should be somewhere in the lab. You do not need to create it on your optical bench.

2. The 100 mm lens (the eyepiece or ocular) and the 200 mm lens (the objective) should be approximately 30 cm apart on the optical bench.

3. Look through the 100 mm lens (toward the 200 mm lens). You can adjust the distance between the lenses until objects across the room are in sharp focus.

Question 3-6: Describe the image you see. Is it upright or inverted? Real or virtual? Magnified?